Experiment No: M5

Experiment Name: Maxwell's Wheel

Objective: Deriving principle of conservation of energy and conversion between kinetic and potential energy.

Keywords: Potential Energy and Linear and Rotation Kinetic Energy.

Theoretical Information:

The Maxwell's wheel experiment is aimed on the investigation of conversions between potential energy and linear and rotation kinetic energy. Physical facts can be classified by the interaction with their environment, as isolated and non-isolated systems. In non-isolated systems, there is energy transfer with the outside of system. However there are no such transfer in isolated systems. Therefore the total energy is conserved in isolated systems.

Energy can be found in different forms (i.e. mechanical, heat, nuclear, chemical, radiation, electrical, etc...) and these forms can transform to each other. As an example, steam engine transforms heat energy to mechanical energy, nuclear reactor transforms nuclear energy to mechanical energy then electrical energy, hydroelectric power plants transforms mechanical energy to electrical energy.

In setup, a rigid metal disk is used with a shaft passing through its middle axis and hanged from the two ends of the shaft by rope. By rolling the ropes on shaft, disk is raised to an upper position and set free to unroll. The disk starts to fall down by rotating around its own axis without any initial velocity. After the rope is fully extended (equilibrium position), disk starts to turn in the opposite direction and starts to roll upward. While the frictions are neglected, the disk should rise to its initial position and continue to this motion periodically. Let's investigate this system closely:

Consider the higher position of disk as $x = h$, and the equilibrium (fully extended) position as $x = 0$. At higher ($x = h$) position ($t = 0$), total energy of system is given by;

$$E = mgx = mgh \tag{5.1}$$

Where m is total mass of disk and shaft?

When the disk loosed free, is starts to roll and move downward, it has a potential energy due to its height ($x = h_x$) a lineer kinetic energy (E_{KL}) due to its movement in vertical direction (v_x) and a rotational kinetic energy (E_{KR}) due to its rotation (w_x).

Where *m* is total mass of disk and shaft. Concisely, the potential energy system had at beginning is transforming to kinetic energy. Due to isolation of system, total energy should be equal to the initial conditions and given by;

$$E_p = mgh_x \text{ , } E_{KL} = \tfrac{1}{2}mv_x^2 \text{ , } E_{KR} = \tfrac{1}{2}Iw_x^2 \tag{5.2}$$

$$E = E_p + E_{KL} + E_{KR} = mgh_x + \frac{1}{2}mv_x^2 + \frac{1}{2}Iw_x^2 = mgh \tag{5.3}$$

If we investigate the equilibrium position; the height is minimum ($x = 0$), lineer and angular velocities have maximum values. However when disk start to rotate in inverse direction and roll up, the kinetic energy transforms to potential energy.

$$E_p = mg0 = 0\ ,\ E_{KL} = \frac{1}{2}mv^2\ ,\ E_{KR} = \frac{1}{2}Iw^2 \qquad 5.4$$

$$E = E_p + E_{KL} + E_{KR} = 0 + \frac{1}{2}mv^2 + \frac{1}{2}Iw^2 = mgh \qquad 5.5$$

The energy of the system can be given by (5.3) at height x=h_x and by (5.1) at height $x = h$. If we could mesaure the rotational period of the disk at a height of $x = h_x$, equation given in (5.3) would have rearranged as;

$$\frac{1}{2}mv_x^2 + \frac{1}{2}Iw_x^2 = mg(h - h_x) \qquad 5.6$$

$$h - h_x = x\ ,\ mgx = \frac{1}{2}mv_x^2 + \frac{1}{2}Iw_x^2 \qquad 5.7$$

Where w is the angular speed of disk and originated from the rolling/unrolling of the rope on shaft.

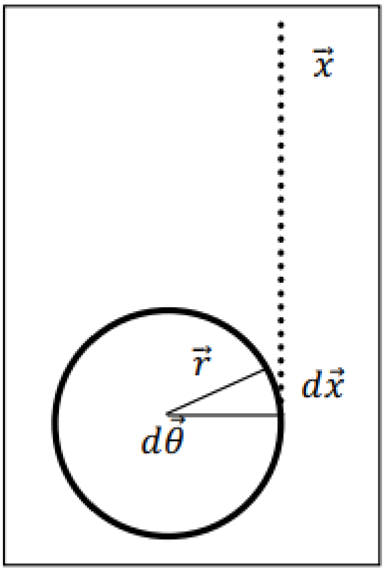

Figure 5.1:

$$d\vec{x} = d\vec{\theta} \times \vec{r} \qquad 5.8$$

$$\vec{v} \equiv \frac{d\vec{x}}{dt} = \frac{d\vec{\theta}}{dt} \times \vec{r} \equiv \vec{w} \times \vec{r} \qquad 5.9$$

$$v = wr \qquad 5.10$$

$$w = \frac{v}{r} \qquad 5.11$$

Here r is the diameter of the shaft. In this case, the total energy equation;

$$mgx = \frac{v_x^2}{2}(m + \frac{I}{r^2}) \qquad 5.12$$

$$gx = \frac{v_x^2}{2}(1 + \frac{I}{mr^2}) \qquad 5.13$$

Moment of inertia of a system consists of two concentric cylinders is the sum of the moment of inertia of both cylinders. Radius is the disk R is more greater than the radius of r, for this case the total moment of inertia can be given by;

$$I = I_{disk} + I_{mil} = \frac{1}{2} m_{disk} R^2 + \frac{1}{2} m_{mil} r^2 \cong \frac{1}{2} m R^2 \quad 5.14$$

m is the total mass of shaft and disk. By using (5.13) and (5.14), speed of the disk passing from x position can be derived as;

$$gx = \frac{v_x^2}{2}\left(1 + \frac{R^2}{2r^2}\right) \quad 5.15$$

$$v_x = \sqrt{\frac{2gx}{\left(1 + \frac{R^2}{2r^2}\right)}} \quad 5.16$$